dtcwt Documentation

Release 0.4.2

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CONTENTS

This library provides support for computing 1D, 2D and 3D dual-tree complex wavelet transforms and their inverse in Python.

The interface is intentionally similar to the existing MATLAB dual-tree complex wavelet transform toolbox provided by Prof. Nick Kingsbury. This library is intended to ease the porting of algorithms written using this toolbox from MATLAB to Python.

The original toolbox is copyrighted and there are some restrictions on use which are outlined in the file `ORIGINAL_README.txt`.

CHAPTER

ONE

TABLE OF CONTENTS

1.1 Getting Started

This section will guide you through installing and using the `dtcwt` library.

1.1.1 Installation

Installation is based on setuptools and follows the usual conventions for a Python project

```
$ python setup.py install
```

A minimal test suite is provided so that you may verify the code works on your system

```
$ python setup.py nosetests
```

This will also write test-coverage information to the `cover/` directory.

1.1.2 Simple usage

Once installed, you are most likely to use one of these functions:

- `dtcwt.dtwavexfm()` – 1D DT-CWT transform.
- `dtcwt.dtwaveifm()` – Inverse 1D DT-CWT transform.
- `dtcwt.dtwavexfm2()` – 2D DT-CWT transform.
- `dtcwt.dtwaveifm2()` – Inverse 2D DT-CWT transform.
- `dtcwt.dtwavexfm3()` – 3D DT-CWT transform.
- `dtcwt.dtwaveifm3()` – Inverse 3D DT-CWT transform.

See *Reference* for full details on how to call these functions. We shall present some simple usage below.

1D transform

This example generates two 1D random walks and demonstrates reconstructing them using the forward and inverse 1D transforms. Note that `dtcwt.dtwavexfm()` and `dtcwt.dtwaveifm()` will transform columns of an input array independently:

```
import numpy as np
from matplotlib.pyplot import *

# Generate a 300x2 array of a random walk
vecs = np.cumsum(np.random.rand(300,2) - 0.5, 0)

# Show input
figure(1)
plot(vecs)
title('Input')

import dtcwt

# 1D transform
Yl, Yh = dtcwt.dtwavexfm(vecs)

# Inverse
vecs_recon = dtcwt.dtwaveifm(Yl, Yh)

# Show output
figure(2)
plot(vecs_recon)
title('Output')

# Show error
figure(3)
plot(vecs_recon - vecs)
title('Reconstruction error')

print('Maximum reconstruction error: {0}'.format(np.max(np.abs(vecs - vecs_recon))))

show()
```

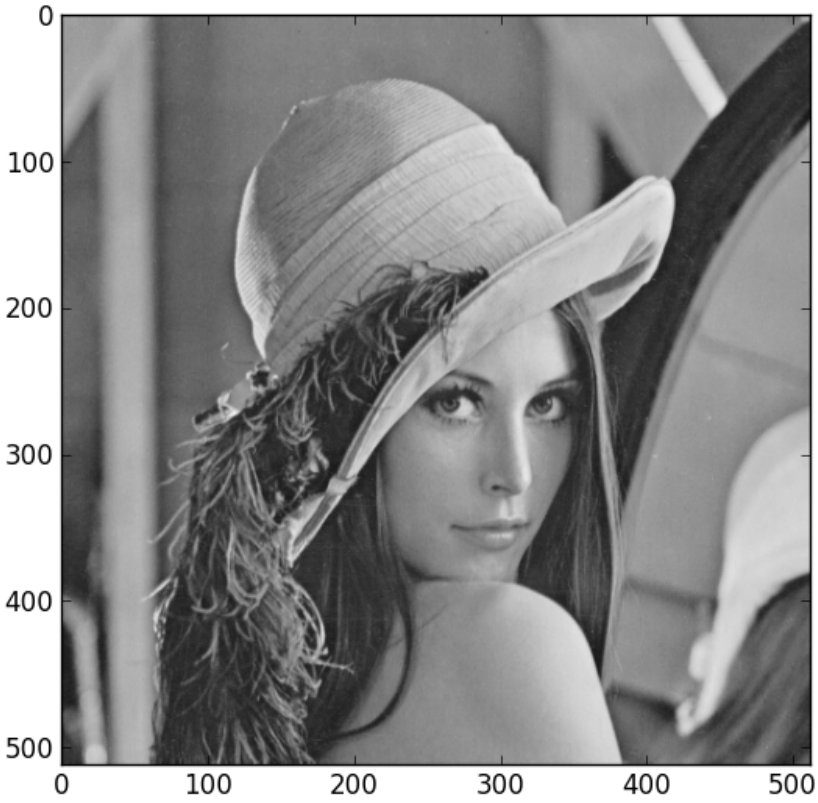
2D transform

Using the pylab environment (part of matplotlib) we can perform a simple example where we transform the standard 'Lena' image and show the level 2 wavelet coefficients:

```
# Load the Lena image from the Internet into a StringIO object
from StringIO import StringIO
from urllib2 import urlopen
LENA_URL = 'http://www.ece.rice.edu/~wakin/images/lena512.pgm'
lena_file = StringIO(urlopen(LENA_URL).read())

# Parse the lena file and rescale to be in the range (0,1]
from scipy.misc import imread
lena = imread(lena_file) / 255.0

from matplotlib.pyplot import *
import numpy as np

# Show lena on the left
figure(1)
imshow(lena, cmap=cm.gray, clim=(0,1))

import dtcwt
```

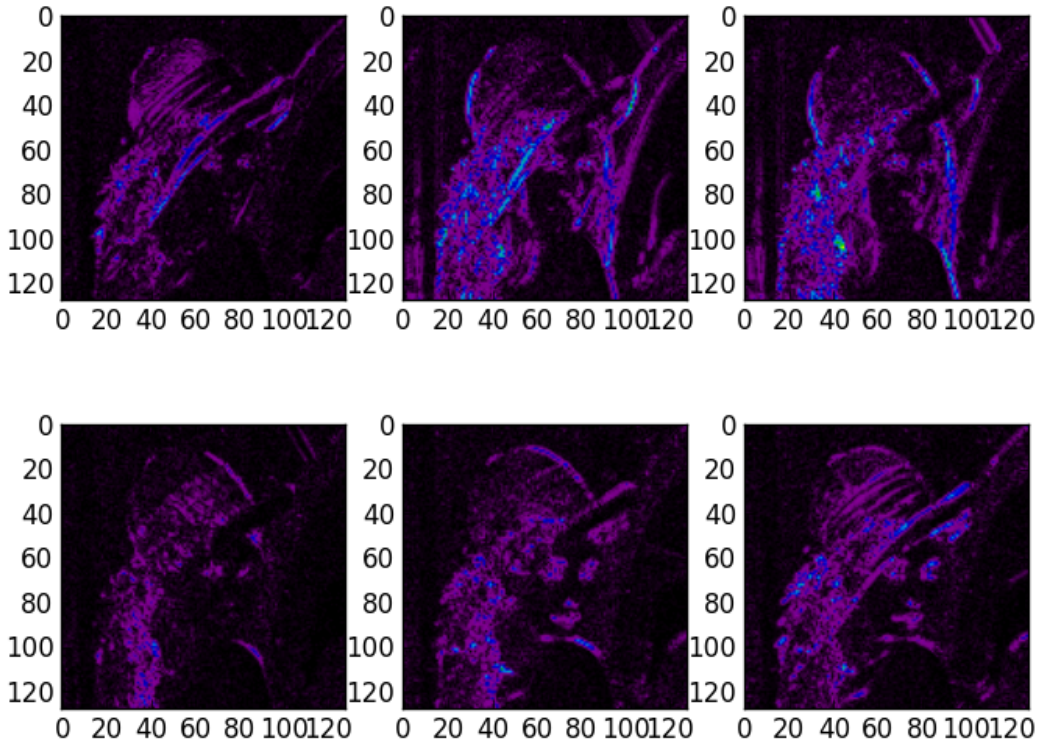
```
# Compute two levels of dtcwt with the defaul wavelet family
Yh, Yl = dtcwt.dtwavexfm2(lena, 2)

# Show the absolute images for each direction in level 2.
# Note that the 2nd level has index 1 since the 1st has index 0.
figure(2)
for slice_idx in xrange(Yl[1].shape[2]):
    subplot(2, 3, slice_idx)
    imshow(np.abs(Yl[1][:,:,slice_idx]), cmap=cm.spectral, clim=(0, 1))

# Show the phase images for each direction in level 2.
figure(3)
for slice_idx in xrange(Yl[1].shape[2]):
    subplot(2, 3, slice_idx)
    imshow(np.angle(Yl[1][:,:,slice_idx]), cmap=cm.hsv, clim=(-np.pi, np.pi))

show()
```

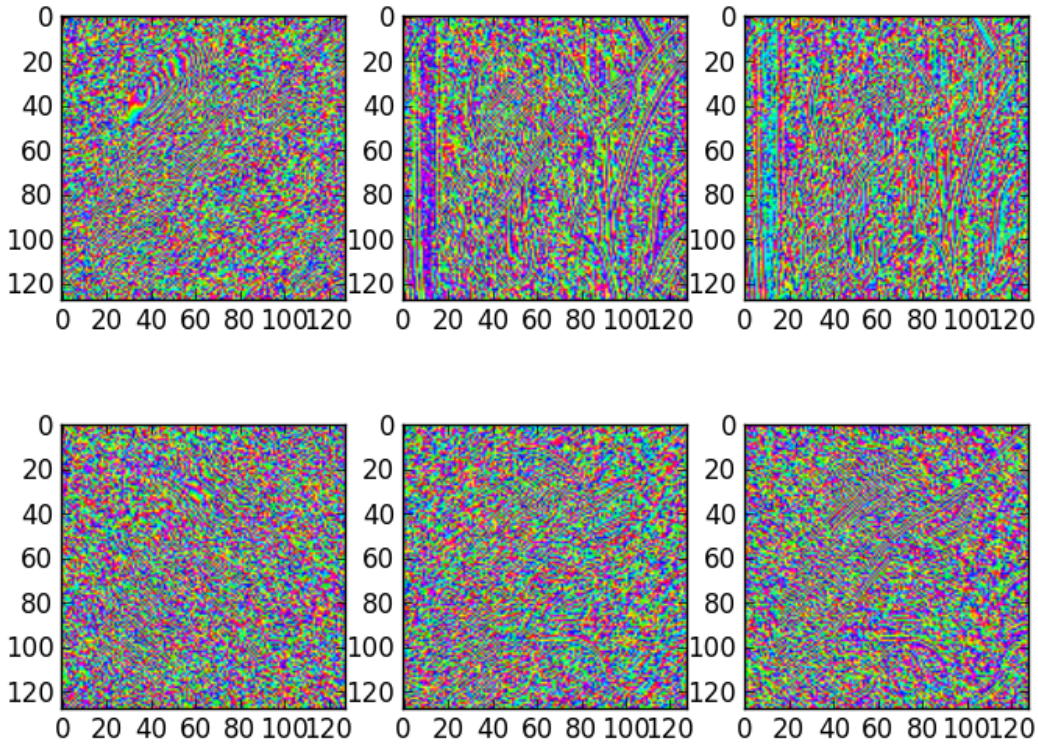
If the library is correctly installed and you also have matplotlib installed, you should see these three figures:



3D transform

In the examples below I assume you've imported pyplot and numpy and, of course, the `dtcwt` library itself:


0
20
40
60
80
100
120
0 20 40 60 80 100120


0
20
40
60
80
100
120
0 20 40 60 80 100120

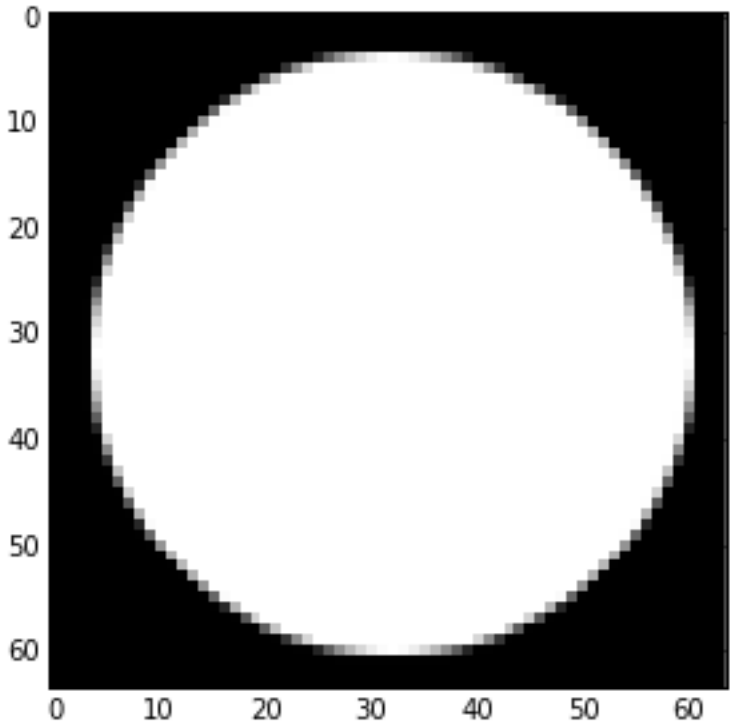
```
import numpy as np
from matplotlib.pyplot import *
from dtcwt import *
```

We can demonstrate the 3D transform by generating a 64x64x64 array which contains the image of a sphere:

```
GRID_SIZE = 64
SPHERE_RAD = int(0.45 * GRID_SIZE) + 0.5

grid = np.arange(-(GRID_SIZE>>1), GRID_SIZE>>1)
X, Y, Z = np.meshgrid(grid, grid, grid)
r = np.sqrt(X*X + Y*Y + Z*Z)

sphere = 0.5 + 0.5 * np.clip(SPHERE_RAD-r, -1, 1)
```

If we look at the central slice of this image, it looks like a circle:

```
imshow(sphere[:,:,GRID_SIZE>>1], interpolation='none', cmap=cm.gray)
```



Performing the 3 level DT-CWT with the defaul wavelet selection is easy:

```
Yl, Yh = dtwavexfm3(sphere, 3)
```

The function returns the lowest level low pass image and a tuple of complex subband coefficients:

```
>>> print(Yl.shape)
(16, 16, 16)
>>> for subbands in Yh:
...     print(subbands.shape)
(32, 32, 32, 28)
(16, 16, 16, 28)
(8, 8, 8, 28)
```

Performing the inverse transform should result in perfect reconstruction:

```
>>> Z = dtwaveifm3(Yl, Yh)
>>> print(np.abs(Z - ellipsoid).max()) # Should be < 1e-12
8.881784197e-15
```

If you plot the locations of the large complex coefficients, you can see the directional sensitivity of the transform:

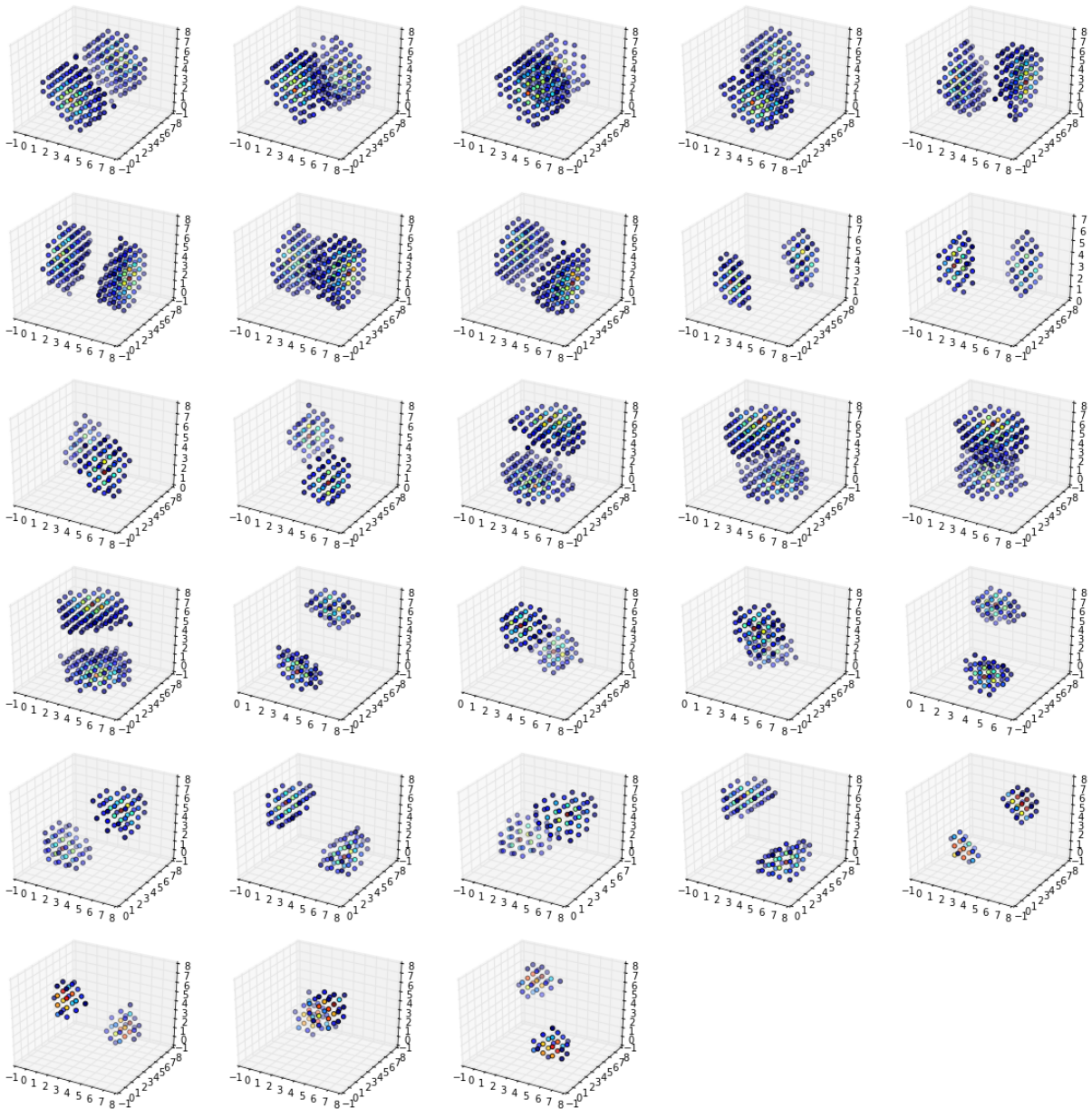
```
from mpl_toolkits.mplot3d import Axes3D

figure(figsize=(16,16))
nplts = Yh[-1].shape[3]
nrows = np.ceil(np.sqrt(nplts))
ncols = np.ceil(nplts / nrows)
for idx in xrange(Yh[-1].shape[3]):
    C = np.abs(Yh[-1][:,:,:,idx])
    ax = gcf().add_subplot(nrows, ncols, idx+1, projection='3d')
    ax.set_aspect('equal')
    x,y,z = np.nonzero(C > 2e-1)
    ax.scatter(x, y, z, c=C[C > 2e-1].ravel())
```

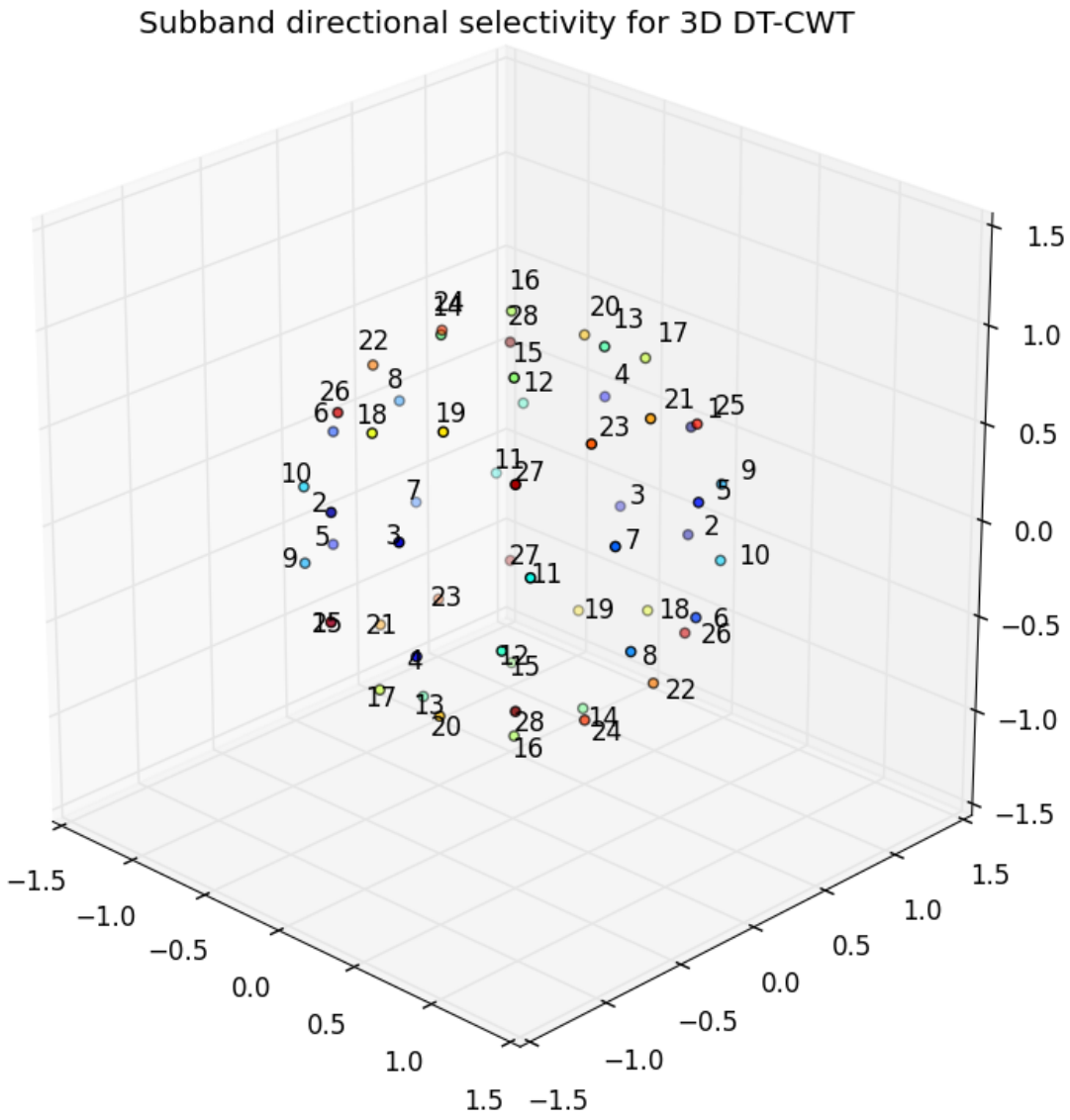
For a further directional sensitivity example, see *Showing 3D Directional Sensitivity*.

1.2 Example scripts

1.2.1 Showing 3D Directional Sensitivity

The `3d_dtcwt_directionality.py` script in the examples directory shows how one may demonstrate the directional sensitivity of the 3D DT-CWT complex subband coefficients. It computes empirically the maximally sensitive directions for each subband and plots them in an interactive figure using matplotlib. A screenshot is reproduced below:





The source for the script is shown below:

```
#!/bin/python

"""
An example of the directional selectivity of 3D DT-CWT coefficients.

This example creates a 3D array holding an image of a sphere and performs the
3D DT-CWT transform on it. The locations of maxima (and their images about the
mid-point of the image) are determined for each complex coefficient at level 2.
These maxima points are then shown on a single plot to demonstrate the
directions in which the 3D DT-CWT transform is selective.

"""
```

```
# Import the libraries we need
from matplotlib.pyplot import *
import numpy as np
from mpl_toolkits.mplot3d import Axes3D
from dtcwt import dtwavexfm3, dtwaveifm3, biort, qshift

# Specify details about sphere and grid size
GRID_SIZE = 128
SPHERE_RAD = 0.33 * GRID_SIZE

# Compute an image of the sphere
grid = np.arange(-(GRID_SIZE>>1), GRID_SIZE>>1)
X, Y, Z = np.meshgrid(grid, grid, grid)
r = np.sqrt(X*X + Y*Y + Z*Z)
sphere = 0.5 + np.clip(SPHERE_RAD-r, -0.5, 0.5)

# Specify number of levels and wavelet family to use
nlevels = 2
b = biort('near_sym_a')
q = qshift('qshift_a')

# Form the DT-CWT of the sphere
Yl, Yh = dtwavexfm3(sphere, nlevels, b, q)

# Plot maxima
figure(figsize=(8,8))

ax = gcf().add_subplot(1,1,1, projection='3d')
ax.set_aspect('equal')
locs = []
scale = 1.1
for idx in xrange(Yh[-1].shape[3]):
    Z = Yh[-1][:,:,:,idx]
    C = np.abs(Z)
    max_loc = np.asarray(np.unravel_index(np.argmax(C), C.shape)) - np.asarray(C.shape)*0.5
    max_loc /= np.sqrt(np.sum(max_loc * max_loc))
    locs.append(max_loc)

    ax.text(max_loc[0] * scale, max_loc[1] * scale, max_loc[2] * scale, str(idx+1))
    ax.text(-max_loc[0] * scale, -max_loc[1] * scale, -max_loc[2] * scale, str(idx+1))

locs = np.asarray(locs)
ax.scatter(locs[:,0], locs[:,1], locs[:,2], c=np.arange(locs.shape[0]))
ax.scatter(-locs[:,0], -locs[:,1], -locs[:,2], c=np.arange(locs.shape[0]))

w = scale * 1.2
ax.auto_scale_xyz([-w, w], [-w, w], [-w, w])

legend()
title('Subband directional selectivity for 3D DT-CWT')
tight_layout()

show()

# vim:sw=4:sts=4:et
```

1.3 Reference

dtcwt.**dtwavexfm**(*X*, *nlevels=3*, *biort='near_sym_a'*, *qshift='qshift_a'*, *include_scale=False*)

Perform a *n*-level DTCWT decompostion on a 1D column vector *X* (or on the columns of a matrix *X*).

Parameters

- **X** – 1D real array or 2D real array whose columns are to be transformed
- **nlevels** – Number of levels of wavelet decomposition
- **biort** – Level 1 wavelets to use. See `biort()`.
- **qshift** – Level >= 2 wavelets to use. See `qshift()`.

Returns Yl The real lowpass image from the final level

Returns Yh A tuple containing the (N, M, 6) shape complex highpass subimages for each level.

Returns Yscale If *include_scale* is True, a tuple containing real lowpass coefficients for every scale.

If *biort* or *qshift* are strings, they are used as an argument to the `biort()` or `qshift()` functions. Otherwise, they are interpreted as tuples of vectors giving filter coefficients. In the *biort* case, this should be (h0o, g0o, h1o, g1o). In the *qshift* case, this should be (h0a, h0b, g0a, g0b, h1a, h1b, g1a, g1b).

Example:

```
# Performs a 5-level transform on the real image X using the 13,19-tap
# filters for level 1 and the Q-shift 14-tap filters for levels >= 2.
Yl, Yh = dtwavexfm(X,5,'near_sym_b','qshift_b')
```

dtcwt.**dtwaveifm**(*Yl*, *Yh*, *biort='near_sym_a'*, *qshift='qshift_a'*, *gain_mask=None*)

Perform an *n*-level dual-tree complex wavelet (DTCWT) 1D reconstruction.

Parameters

- **Yl** – The real lowpass subband from the final level
- **Yh** – A sequence containing the complex highpass subband for each level.
- **biort** – Level 1 wavelets to use. See `biort()`.
- **qshift** – Level >= 2 wavelets to use. See `qshift()`.
- **gain_mask** – Gain to be applied to each subband.

Returns Z Reconstructed real array.

The *l*-th element of *gain_mask* is gain for wavelet subband at level l. If gain_mask[l] == 0, no computation is performed for band *l*. Default *gain_mask* is all ones. Note that *l* is 0-indexed.

If *biort* or *qshift* are strings, they are used as an argument to the `biort()` or `qshift()` functions. Otherwise, they are interpreted as tuples of vectors giving filter coefficients. In the *biort* case, this should be (h0o, g0o, h1o, g1o). In the *qshift* case, this should be (h0a, h0b, g0a, g0b, h1a, h1b, g1a, g1b).

Example:

```
# Performs a reconstruction from Yl,Yh using the 13,19-tap filters
# for level 1 and the Q-shift 14-tap filters for levels >= 2.
Z = dtwaveifm(Yl, Yh, 'near_sym_b', 'qshift_b')
```

dtcwt.**dtwavexfm2**(*X*, *nlevels=3*, *biort='near_sym_a'*, *qshift='qshift_a'*, *include_scale=False*)

Perform a *n*-level DTCWT-2D decompostion on a 2D matrix *X*.

Parameters

- **X** – 2D real array
- **nlevels** – Number of levels of wavelet decomposition
- **biort** – Level 1 wavelets to use. See `biort()`.
- **qshift** – Level >= 2 wavelets to use. See `qshift()`.

Returns Yl The real lowpass image from the final level

Returns Yh A tuple containing the complex highpass subimages for each level.

Returns Yscale If *include_scale* is True, a tuple containing real lowpass coefficients for every scale.

If *biort* or *qshift* are strings, they are used as an argument to the `biort()` or `qshift()` functions. Otherwise, they are interpreted as tuples of vectors giving filter coefficients. In the *biort* case, this should be (h0o, g0o, h1o, g1o). In the *qshift* case, this should be (h0a, h0b, g0a, g0b, h1a, h1b, g1a, g1b).

Example:

```
# Performs a 3-level transform on the real image X using the 13,19-tap
# filters for level 1 and the Q-shift 14-tap filters for levels >= 2.
Yl, Yh = dtwavexfm2(X, 3, 'near_sym_b', 'qshift_b')
```

dtcwt.**dtwaveifm2**(*Yl*, *Yh*, *biort='near_sym_a'*, *qshift='qshift_a'*, *gain_mask=None*)

Perform an *n*-level dual-tree complex wavelet (DTCWT) 2D reconstruction.

Parameters

- **Yl** – The real lowpass subband from the final level
- **Yh** – A sequence containing the complex highpass subband for each level.
- **biort** – Level 1 wavelets to use. See `biort()`.
- **qshift** – Level >= 2 wavelets to use. See `qshift()`.
- **gain_mask** – Gain to be applied to each subband.

Returns Z Reconstructed real array

The (*d*, *l*)-th element of *gain_mask* is gain for subband with direction *d* at level *l*. If gain_mask[d,l] == 0, no computation is performed for band (d,l). Default *gain_mask* is all ones. Note that both *d* and *l* are zero-indexed.

If *biort* or *qshift* are strings, they are used as an argument to the `biort()` or `qshift()` functions. Otherwise, they are interpreted as tuples of vectors giving filter coefficients. In the *biort* case, this should be (h0o, g0o, h1o, g1o). In the *qshift* case, this should be (h0a, h0b, g0a, g0b, h1a, h1b, g1a, g1b).

Example:

```
# Performs a 3-level reconstruction from Yl,Yh using the 13,19-tap
# filters for level 1 and the Q-shift 14-tap filters for levels >= 2.
Z = dtwaveifm2(Yl, Yh, 'near_sym_b', 'qshift_b')
```

dtcwt.**dtwavexfm3**(*X*, *nlevels=3*, *biort='near_sym_a'*, *qshift='qshift_a'*, *ext_mode=4*)

Perform a *n*-level DTCWT-3D decompostion on a 3D matrix *X*.

Parameters

- **X** – 3D real array-like object
- **nlevels** – Number of levels of wavelet decomposition
- **biort** – Level 1 wavelets to use. See `biort()`.
- **qshift** – Level >= 2 wavelets to use. See `qshift()`.

- **ext_mode** – Extension mode. See below.

Returns Yl The real lowpass image from the final level

Returns Yh A tuple containing the complex highpass subimages for each level.

Each element of *Yh* is a 4D complex array with the 4th dimension having size 28. The 3D slice `Yh[l][:,:,:,d]` corresponds to the complex higpass coefficients for direction d at level l where d and l are both 0-indexed.

If *biort* or *qshift* are strings, they are used as an argument to the `biort()` or `qshift()` functions. Otherwise, they are interpreted as tuples of vectors giving filter coefficients. In the *biort* case, this should be (h0o, g0o, h1o, g1o). In the *qshift* case, this should be (h0a, h0b, g0a, g0b, h1a, h1b, g1a, g1b).

There are two values for *ext_mode*, either 4 or 8. If *ext_mode* = 4, check whether 1st level is divisible by 2 (if not we raise a `ValueError`). Also check whether from 2nd level onwards, the coefs can be divided by 4. If any dimension size is not a multiple of 4, append extra coefs by repeating the edges. If *ext_mode* = 8, check whether 1st level is divisible by 4 (if not we raise a `ValueError`). Also check whether from 2nd level onwards, the coeffs can be divided by 8. If any dimension size is not a multiple of 8, append extra coeffs by repeating the edges twice.

Example:

```
# Performs a 3-level transform on the real 3D array X using the 13,19-tap
# filters for level 1 and the Q-shift 14-tap filters for levels >= 2.
Yl, Yh = dtwavexfm3(X, 3, 'near_sym_b', 'qshift_b')
```

dtcwt.**dtwaveifm3**(*Yl*, *Yh*, *biort='near_sym_a'*, *qshift='qshift_a'*, *ext_mode=4*)

Perform an *n*-level dual-tree complex wavelet (DTCWT) 3D reconstruction.

Parameters

- **Yl** – The real lowpass subband from the final level
- **Yh** – A sequence containing the complex highpass subband for each level.
- **biort** – Level 1 wavelets to use. See `biort()`.
- **qshift** – Level >= 2 wavelets to use. See `qshift()`.
- **ext_mode** – Extension mode. See below.

Returns Z Reconstructed real image matrix.

If *biort* or *qshift* are strings, they are used as an argument to the `biort()` or `qshift()` functions. Otherwise, they are interpreted as tuples of vectors giving filter coefficients. In the *biort* case, this should be (h0o, g0o, h1o, g1o). In the *qshift* case, this should be (h0a, h0b, g0a, g0b, h1a, h1b, g1a, g1b).

There are two values for *ext_mode*, either 4 or 8. If *ext_mode* = 4, check whether 1st level is divisible by 2 (if not we raise a `ValueError`). Also check whether from 2nd level onwards, the coefs can be divided by 4. If any dimension size is not a multiple of 4, append extra coefs by repeating the edges. If *ext_mode* = 8, check whether 1st level is divisible by 4 (if not we raise a `ValueError`). Also check whether from 2nd level onwards, the coeffs can be divided by 8. If any dimension size is not a multiple of 8, append extra coeffs by repeating the edges twice.

Example:

```
# Performs a 3-level reconstruction from Yl,Yh using the 13,19-tap
# filters for level 1 and the Q-shift 14-tap filters for levels >= 2.
Z = dtwaveifm3(Yl, Yh, 'near_sym_b', 'qshift_b')
```

dtcwt.**biort**(*name*)

Load level 1 wavelet by name.

Parameters **name** – a string specifying the wavelet family name

Returns a tuple of vectors giving filter coefficients

Name	Wavelet
antonini	Antonini 9,7 tap filters.
legall	LeGall 5,3 tap filters.
near_sym_a	Near-Symmetric 5,7 tap filters.
near_sym_b	Near-Symmetric 13,19 tap filters.

Return a tuple whose elements are a vector specifying the h0o, g0o, h1o and g1o coefficients.

Raises

- **IOError** – if name does not correspond to a set of wavelets known to the library.
- **ValueError** – if name specifies a `qshift()` wavelet.

`dtcwt.`**`qshift`**(*name*)

Load level >=2 wavelet by name,

Parameters **name** – a string specifying the wavelet family name

Returns a tuple of vectors giving filter coefficients

Name	Wavelet
qshift_06	Quarter Sample Shift Orthogonal (Q-Shift) 10,10 tap filters, (only 6,6 non-zero taps).
qshift_a	Q-shift 10,10 tap filters, (with 10,10 non-zero taps, unlike qshift_06).
qshift_b	Q-Shift 14,14 tap filters.
qshift_c	Q-Shift 16,16 tap filters.
qshift_d	Q-Shift 18,18 tap filters.

Return a tuple whose elements are a vector specifying the h0a, h0b, g0a, g0b, h1a, h1b, g1a and g1b coefficients.

Raises

- **IOError** – if name does not correspond to a set of wavelets known to the library.
- **ValueError** – if name specifies a `biort()` wavelet.

1.3.1 Low-level support functions

A normal user should not need to call these functions but they are documented here just in case you do.

`dtcwt.lowlevel.`**`as_column_vector`**(*v*)

Return *v* as a column vector with shape (N,1).

`dtcwt.lowlevel.`**`coldfilt`**(*X*, *ha*, *hb*)

Filter the columns of image X using the two filters ha and hb = reverse(ha). ha operates on the odd samples of X and hb on the even samples. Both filters should be even length, and h should be approx linear phase with a quarter sample advance from its mid pt (i.e. $|h(m/2)| > |h(m/2+1)|$).

```
                  ext         top edge                      bottom edge       ext
Level 1:        !               |               !               |               !
odd filt on .    b   b   b   b   a   a   a   a   a   a   a   a   b   b   b   b
odd filt on .      a   a   a   a   b   b   b   b   b   b   b   b   a   a   a   a
Level 2:        !               |               !               |               !
+q filt on x      b       b       a       a       a       a       b       b
-q filt on o          a       a       b       b       b       b       a       a
```

The output is decimated by two from the input sample rate and the results from the two filters, Ya and Yb, are interleaved to give Y. Symmetric extension with repeated end samples is used on the composite X columns before each filter is applied.

Raises ValueError if the number of rows in X is not a multiple of 4, the length of ha does not match hb or the lengths of ha or hb are non-even.

dtcwt.lowlevel.**colfilter**(*X*, *h*)

Filter the columns of image *X* using filter vector *h*, without decimation. If len(h) is odd, each output sample is aligned with each input sample and *Y* is the same size as *X*. If len(h) is even, each output sample is aligned with the mid point of each pair of input samples, and Y.shape = X.shape + [1 0].

Parameters

- **X** – an image whose columns are to be filtered
- **h** – the filter coefficients.

Returns Y the filtered image.

dtcwt.lowlevel.**colifilt**(*X*, *ha*, *hb*)

Filter the columns of image X using the two filters ha and hb = reverse(ha). ha operates on the odd samples of X and hb on the even samples. Both filters should be even length, and h should be approx linear phase with a quarter sample advance from its mid pt (i.e *:math:'|h(m/2)| > |h(m/2 + 1)|*).

```
                  ext       left edge                       right edge       ext
Level 2:        !               |               !               |               !
+q filt on x      b       b       a       a       a       a       b       b
-q filt on o          a       a       b       b       b       b       a       a
Level 1:        !               |               !               |               !
odd filt on .    b   b   b   b   a   a   a   a   a   a   a   a   b   b   b   b
odd filt on .      a   a   a   a   b   b   b   b   b   b   b   b   a   a   a   a
```

The output is interpolated by two from the input sample rate and the results from the two filters, Ya and Yb, are interleaved to give Y. Symmetric extension with repeated end samples is used on the composite X columns before each filter is applied.

dtcwt.lowlevel.**reflect**(*x*, *minx*, *maxx*)

Reflect the values in matrix *x* about the scalar values *minx* and *maxx*. Hence a vector *x* containing a long linearly increasing series is converted into a waveform which ramps linearly up and down between *minx* and *maxx*. If *x* contains integers and *minx* and *maxx* are (integers + 0.5), the ramps will have repeated max and min samples.

CHAPTER

TWO

INDICES AND TABLES

- *genindex*
- *modindex*
- *search*

PYTHON MODULE INDEX

d

dtcwt, ??
dtcwt.lowlevel, ??